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Experimental control of chaos in a delayed high-dimensional system

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Abstract. We introduce a novel control method for a delayed dynamical system exhibiting high-dimensional chaos. The control is based on a negative feedback loop with an adaptive filtering, consisting of a selective filter, centered at the frequency of the orbit to be stabilized, with the addition of a time derivative correction. The validity of the method is also discussed in the framework of a space-like representation adopted to study the analogies between delayed dynamical systems and spatially extended systems.

PACS. 05.45.-a Nonlinear dynamics and nonlinear dynamical systems – 42.50.Lc Quantum fluctuations, quantum noise, and quantum jumps – 42.55.Lt Gas lasers including excimer and metal-vapor lasers

1 Introduction

Spatiotemporal chaos, *i.e.* the presence of deterministic chaos in spatially extended systems, has been largely investigated and discussed in nonlinear optics [1]. To this aim, order parameter equations, in the form of Ginzburg-Landau [2,3] or Swift-Hohenberg equations [4,5], have been used to interpret experimental results in several pattern forming devices. The interest in nonlinear optics is also related to the fact that the coupling between diffraction or dispersion and medium nonlinearities may be the source of spatial instabilities.

Stabilizing spatiotemporal chaos into a regular periodic behavior is relevant for many applications. The chaos control method by Ott, Grebogi and Yorke [6] consists in stabilizing one of the unstable periodic orbits embedded in a chaotic attractor by perturbing a control parameter. Development of chaos control techniques [7] to spatially extended media for stabilizing patterns and manipulating spatiotemporal dynamics [8] is nowadays a major challenge, and different schemes have been demonstrated [9].

Here we present an efficient control method for a delayed feedback system. These systems, for delays long with respect to the intrinsic decorrelation time corresponding to a short-delay dynamics, display a high-dimension chaotic behavior. A suitable space-like representation [10] provides an analogy between delayed systems and space extended systems. The analogy rests on the fact that the time variable t can be decomposed into a continuous spacelike variable σ $(0 \leq \sigma \leq \tau)$ and a discrete timelike variable $n\tau$:

$$t \equiv \sigma + n\tau \,,$$

where τ is the delay time. In this framework, the long-range temporal interaction due to the delayed feedback can be considered as a local interaction from one to the next delay unit. Thus, the behavior of such a system can be studied by representing the original signal $x(t)$ as a space-time plot of a one-dimensional spatial system of length $L = \tau$ on a discrete-time lattice. Typically, when the delay τ is larger than the characteristic oscillating period, it is possible to observe phase defects, *i.e.* points where the phase presents a discontinuity and the amplitude goes to zero [11].

Here we refer to the pseudo-spatial structures produced by a CO_2 laser with delayed feedback. In particular we demonstrate the possibility of controlling irregular patterns by applying a real-time implementation of an adaptive algorithm [12]. This method allows stabilization of the period and the amplitude of the chaotic oscillations by eliminating the zero-amplitude points which can originate spatial defects [13].

2 The experimental set-up and results

The experimental system is a single-mode CO_2 laser with an electro-optic feedback on the cavity losses (Fig. 1). A photodetector yields a voltage proportional to the laser output intensity. This voltage, after a suitable delay and amplification, drives an intracavity electro-optic modulator. The detector is a fast Hg-Cd-Te photodiode and the delay line is realized using fast (2 MHz) and accurate (12 bits) A/D (analog-to-digital) and D/A converters allowing variations of the delay time τ up to 130 ms with 0.5 μs

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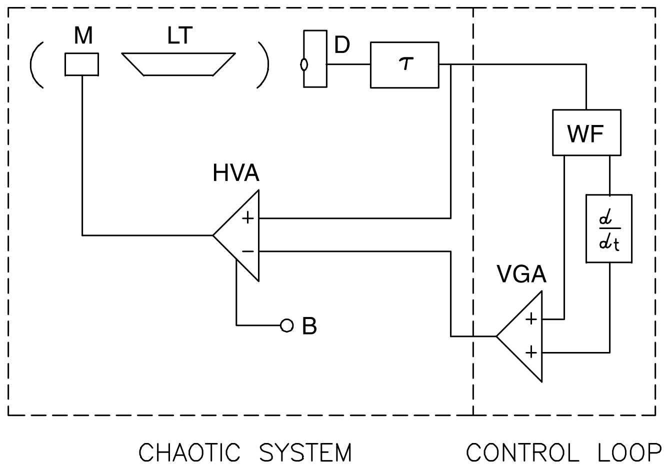

Fig. 1. Experimental setup. M: electro-optic modulator; LT: laser tube; D: HgCdTe detector; τ: delay line; WF: washout filter; d/dt: derivative block; VGA: variable gain amplifier; HVA: high-voltage amplifier; B: bias input.

resolution. The high-voltage amplifier adds to the delayed signal a continuous voltage level B which, once τ is fixed, acts as the control parameter.

Even at zero delay, by increasing the bias B, the system undergoes a Hopf bifurcation, with the fundamental period around 20 μs, and eventually it enters a chaotic regime. If the delay τ is of the order of the oscillating period of the system, the fractal dimension of the chaotic attractor, reached through quasiperiodicity, remains around 3 [14]. Since the aim of the present work is to implement a control strategy for a high-dimensional chaotic regime (characterized by more than one positive Lyapunov exponent), we have explored a delay range from $\tau = 50$ μs to $\tau = 600$ μs. By a simple argument [14], the number of extra degrees of freedom due to the delay is given approximately by the ratio between τ and the correlation time of the intrinsic chaotic motion which is around 100 μs. A rigorous demonstration of the high dimensionality of the system requires a relevant effort, not only because a large number of data points have to be analyzed, but also because new techniques are necessary, since in this situation traditional methods, such as correlation dimension evaluation by the Grassberger-Procaccia algorithm, do not provide consistent results [15]. Evidence that delayed systems such as ours can have more than one positive Lyapunov exponent for τ larger than 50 μs has been recently given with a new method of analyzing delayed dynamics [15, 16]. For the above range of τ values and for increasing B, the system always shows the same qualitative behavior. Superimposed to the Hopf oscillation, we observe a deep modulation paced by the inverse of the delay τ, while the attractor becomes chaotic. A typical time sequence of the laser intensity $x(t)$ is shown in Figure 2 for $\tau = 150$ μs, together with the corresponding power spectrum. For such a delay time, four positive Lyapunov exponents have been found [16], with a Kaplan-Yorke dimension of about 8.5. A further increase of B leads to a collapse of the chaotic attractor into another stable limit cycle. The signal ac-

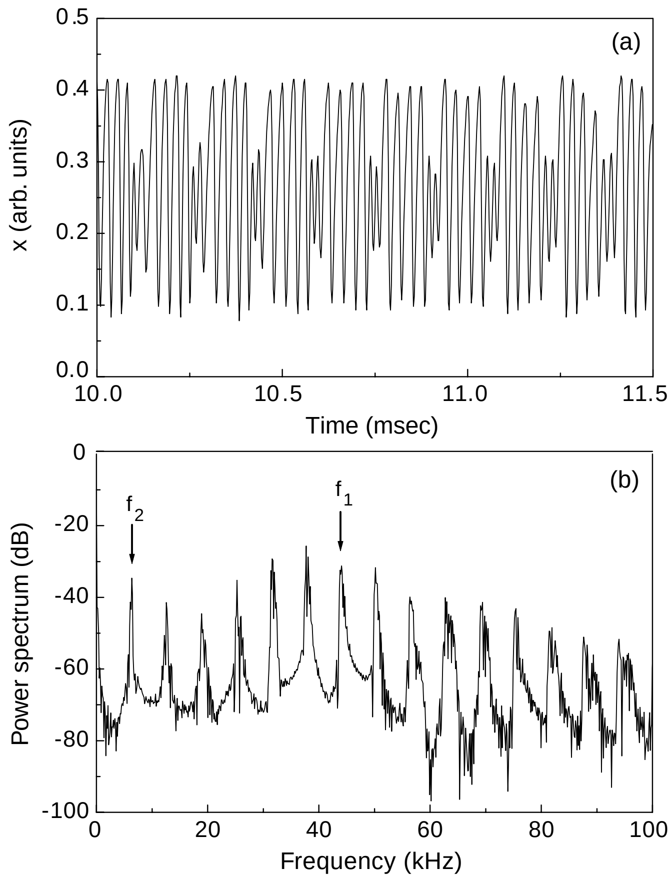

Fig. 2. (a) Laser intensity signal for $\tau = 150$ μs and $B = 275$ V. (b) Corresponding power spectrum. $f_1 = 1/T = 43$ kHz is the Hopf frequency, $f_2 = 1/\tau = 6.67$ kHz is the reciprocal delay time.

quisition was performed by means of a 12-bits PCI card (National Instruments PCI-MIO-16E-1), driven by a custom software application developed with LabView, that performs also a real-time 2D reconstruction of the signal. Figure 3 shows such a representation at the same parameter values corresponding to Figure 2. The aperiodic behavior of the system is clearly proved by the presence of irregular roll structures. Similar results can be obtained for the other τ values previously specified.

The first step to achieve stabilization is to employ a "washout filter" [17]. It consists of a negative feedback loop configuration, wherein all unwanted frequencies present in the chaotic spectrum of the output signal $x(t)$ are transmitted as corrections through a selective filter. In this way, the system is allowed to oscillate at the only frequency which is not subtracted, namely that of the unstable orbit to be stabilized. The observation of the power spectrum reported in Figure 2 can help in defining the filter characteristics. The peak structure is due to the competition between the Hopf frequency $f_1 = 43$ kHz and the inverse of the delay time $1/\tau = f_2 = 6.67$ kHz. Thus, following a strategy already tested [18], we use a washout filter whose transfer function presents a zero of amplitude at f_1 and a zero of phase at f_2 (see Fig. 4). In this way,

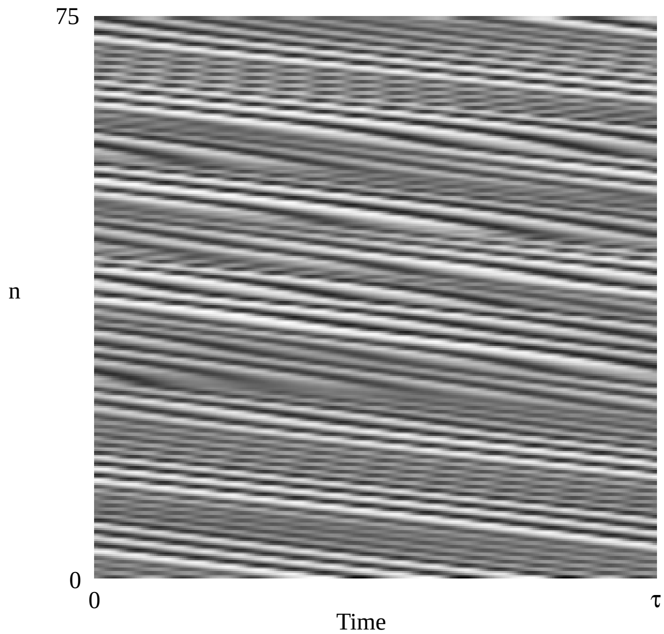

Fig. 3. Two-dimensional plot of the experimental signal of Figure 2. The horizontal axis represents time (the full scale is equal to a delay time $\tau = 150$ μs). On the vertical axis 75 consecutive delay units are reported.

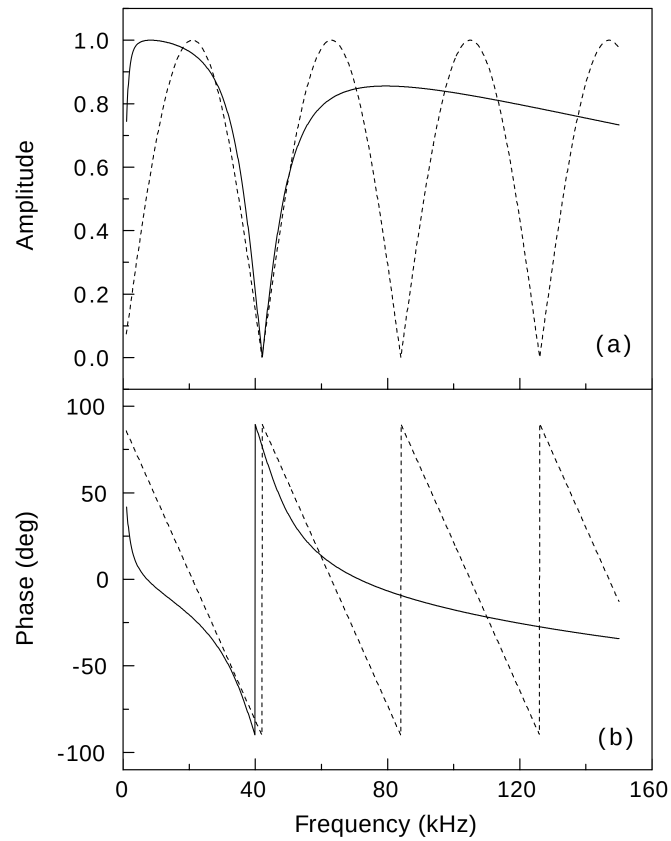

Fig. 4. Amplitude and phase of the two transfer functions $C(s)$ (solid line) and $C'(s)$ (dashed line). For both functions $f_1 = 1/T = 43$ kHz.

the feedback loop provides minimum and maximum rejection at f_1 and f_2, respectively. These features could in principle stabilize the unstable limit cycle with frequency f_1, while canceling the effects of the delay at frequency f_2. The transfer function of Figure 4 can be represented analitically by the following formula ($s = i\omega$) [18]

$$C(s) = \frac{ks(s^2 + \Omega^2)}{\left(s^2 + \zeta\Omega s + \dfrac{\Omega^2}{4}\right)(s + \mu\Omega)},$$

where $\Omega = 2\pi f_1$, k is the gain, ζ and μ suitable parameters.

It is important to observe that the above control method presents several analogies with the Time Delayed Auto-Synchronization (TDAS) method, proposed by Pyragas [19]. The TDAS consists in a feedback which reinjects into the system the difference between the output signal $x(t)$ and its delayed value $x(t - T)$ with a certain amplification factor K; this can allow stabilization of an unstable orbit with period T. The control signal is in the form

$$U(t) = K[x(t - T) - x(t)],$$

and the corresponding transfer function is

$$C'(s) = K(1 - e^{-sT}).$$

In the low-frequency domain, the transfer function $C'(s)$ is similar to that of the washout filter $C(s)$ [17], since both present a zero of the amplitude at f_1, provided $f_1 = 1/T$ (Fig. 4). On the other hand, the TDAS transfer function amplitude goes to zero also for all f_1 harmonics (thus ensuring exact zeroing of the control signal when the stabilization is obtained), while the phase necessarily crosses zero at $f_1/2$. Several experimental applications of the TDAS method have been reported in the literature [20]. Recently, Just *et al.* [21] analyzed the mechanism of time-delayed feedback control from a theoretical point of view, and their expectations have been verified in a nonlinear electronic circuit. The same authors also addressed the problem of the mismatch between the delay τ and the period T of the unstable orbit to be stabilized, and the influence of the control loop latency [22].

At variance with the results on low-dimensional chaotic dynamics, the application of a control loop based only on the washout filter fails when $\tau > 50$ μs. The presence of a large delay and, consequently, of high-dimensional chaotic signals suggests that more information on the state of the system is needed in the control loop in order to achieve stabilization. For this reason, we have studied the possibility of implementing an analog version of the adaptive algorithm (AA) proposed in [12]. It consists in a sequence of variable resolution observation intervals at which the state variable is sampled. The sampling times are chosen so that the sequence of observed points forms a regularized set, in the sense that the separation of adjacent points is almost uniform. This technique

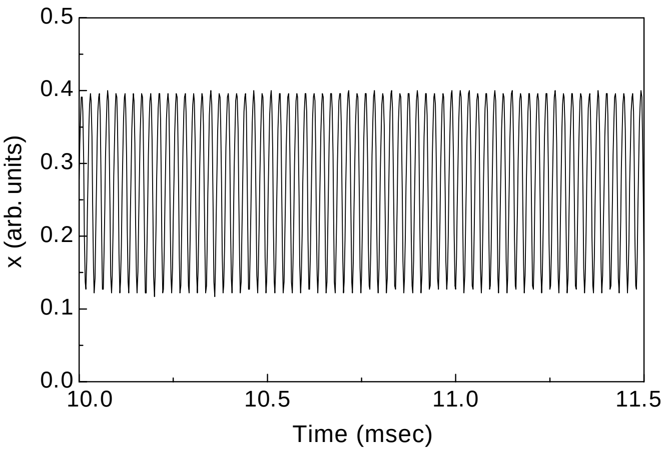

Fig. 5. Stabilized Hopf oscillation of the laser intensity for the same parameter values of Figure 2.

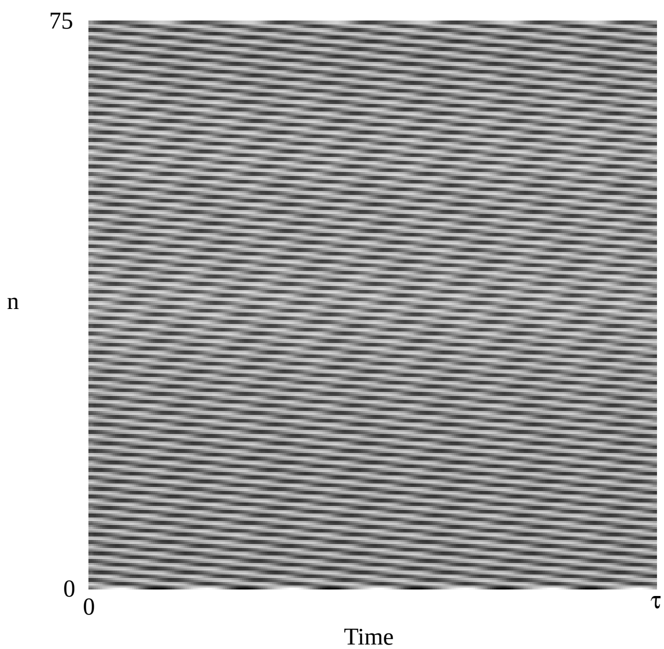

Fig. 6. Two-dimensional plot of the experimental signal of Figure 5.

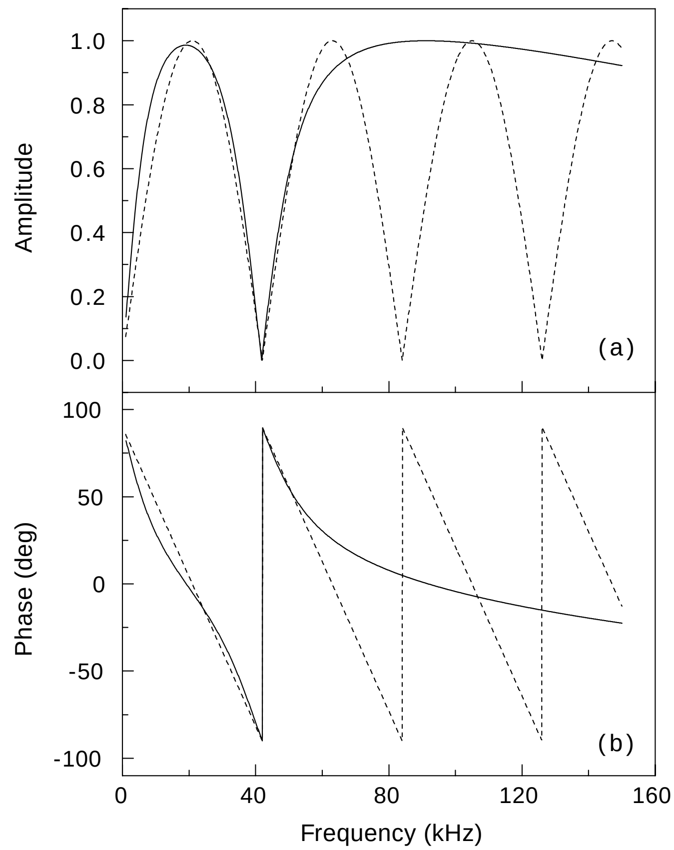

Fig. 7. Amplitude and phase of the second washout filter (solid line) compared with $C'(s)$ (dashed line). For both functions $f_1 = 1/T = 43$ kHz.

would require a computer-aided real-time analysis of the temporal signal. Anyway, it has been demonstrated that the AA can be conveniently approximated at the first order in time with the following control signal (dots denote time derivatives) [13]:

$$U(t) = K_1[x(t-T) - x(t)] + K_2[\dot{x}(t-T) - \dot{x}(t)]$$

which, considering the close analogy between $C(s)$ and $C'(s)$, leads to the following transfer function:

$$C^*(s) = (K_1 + sK_2)C(s)\,.$$

Such a control can be easily realized if the output voltage from the washout filter is added with its time derivative (see Fig. 1). Obviously, the amplification factors for the two signals contributing to the global control must be regulated separately.

With the use of this improved scheme we have converted the chaotic motion into a limit cycle with period $1/f_1$; the time sequence of the stabilized output laser intensity and its 2D representation are reported in Figures 5 and 6, respectively, for the same parameter values of Figures 2 and 3. It is important to stress the following experimental results: *i*) the control signal compared with the unperturbed delayed feedback is of the order of a few percent; *ii*) the control works for $K_2 \geq 0.8\,K_1$ thus confirming the relevance of the derivative correction. For $K_2 < 0.8\,K_1$, long delays are not stabilized by the washout filter. In such a case, an increase of the amount of perturbation K_1 above a few percent changes drastically the nature of the dynamics, forcing the system into another periodic orbit no longer coinciding with the original Hopf orbit; *iii*) the control loop has been entirely realized with analog circuitry and consequently can be very fast; *iv*) finally, stabilization is effective independently of the delay time τ. This last observation gave us hint to verify that the condition of having a zero in the phase diagram of $C(s)$ at $f_2 = 1/\tau$ can be relaxed. In fact, we have again obtained stabilization for all τ values after replacing the washout filter with another filter having a transfer function closer to $C'(s)$ (see Fig. 7).

3 Conclusions

We have shown that in a delayed dynamical system until delays are relatively short (*i.e.* comparable with the decorrelation time of the nondelayed chaotic system), it is possible to stabilize periodic behaviours by using a feedback control based on selective filtering. This selective filtering displays the same essential features as a TDAS recently explored in detail [22]. However, for a long delay inducing a high-dimensional chaos, it is necessary to recur to an adaptive control, based on the combination of a washout filter with its time derivative, in a negative feedback configuration.

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